Eur. Phys. J. D **29**, 279–281 (2004)
DOI: 10.1140/epjd/e2004-00054-x

THE EUROPEAN PHYSICAL JOURNAL D

Comment on

Kadomtsev-Petviashvili (KP) Burgers equation in a dusty plasmas with non-adiabatic dust charge fluctuation by J.-K. Xue

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Eur. Phys. J. D **26**, 211 (2003)

Received 23 November 2003
Published online 20 April 2004 –

Abstract. We hereby present several comments on reference [1]: for this model, we present the travelling shock-wave solutions different from that of reference [1], an auto-Bäcklund transformation, and several two-dimensional and non-travelling-wave possibly-observable effects for the future space and laboratory plasma experiments.

PACS. 52.35.Sb Solitons; BGK modes – 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.)

1 Background

The observations on the ion-acoustic shocks in dusty plasmas [2–5] have been explained via the one-dimensional (nonintegrable) Korteweg-de Vries-Burgers equation (1D-KdVB) [5,6] (and references therein), with the monotonic shock implying that the Burgers term is strong enough. For an unmagnetized dusty plasma containing immobile charged dust grains, warm electrons and warm ions, reference [7] briefly states that the 1D-KdVB admits both oscillatory and monotonic shocks. The exact analytic expression for the monotonic shock has been given, e.g., in references [6,8] and references therein, but the exact analytic expression for the oscillatory shock hasn't been reported yet.

For the space and laboratory plasma systems, the investigations on the two-dimensional Korteweg-de Vries-Burgers equation have also been performed [9],

$$(u_t + uu_x + u_{xxx} + fu_{xx})_x + cu_{yy} = 0, \qquad (1)$$

which is also called the Kadomtsev-Petviashvili-Burgers equation [1],

$$(\psi_\tau + A\psi\psi_\xi + B\psi_{\xi\xi\xi} - C\psi_{\xi\xi})_\xi + D\psi_{\eta\eta} = 0. \qquad (2)$$

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It is easy to see that equations (1, 2) are the same, by virtue of the scaling transformations: $x = B^{-1/3}\xi$, $y = \eta$, $t = \tau$, $u(x, y, t) = AB^{-1/3}\psi(\xi, \eta, \tau)$, $f = -CB^{-2/3}$ and $c = DB^{1/3}$. The physical implications of the variables t, x, y, of the real constants f, c, as well as of the function $u(x, y, t)$ are given by the plasma system under investigation [1,3,5,9].

Of current importance, reference [1] has studied the nonlinear dust acoustic waves in the dusty plasmas with the combined effects of the non-adiabatic dust charge fluctuation and higher-order transverse perturbation, with equation (2) describing the dust acoustic waves, with a particular travelling-shock-wave solution obtained.

Other recent relevant work can be found in, e.g., references [10–19].

2 Comment 1

Since the particular travelling-shock-wave solution in reference [1], i.e., expression (44) there, seems questionable, we make use of the Wu elimination method, which is a very sufficient method to solve for the systems of algebraic polynomial equations with many unknowns [12,20,21], as well as the auto-Bäcklund transformation given below, so as to hereby calculate out the following travelling shock-wave solutions for equation (2), which is different from expression (44) of reference [1],

$$\psi(\xi,\eta,\tau)=\frac{6C^2\kappa}{25AB}+\frac{B^{\frac{1}{3}}u_2}{A}+\frac{3C^2}{25AB}\mathrm{sech}^2\left[\frac{\alpha_4}{2}+\frac{\alpha_2\eta}{2}+\left(\frac{3C^3}{125B^2}+\frac{5{\alpha_2}^2BD\kappa}{2C}+\frac{C\kappa u_2}{10B^{\frac{2}{3}}}\right)\tau-\frac{C\kappa\xi}{10B}\right]+\frac{6C^2\kappa}{25AB}\tanh\left[\frac{\alpha_4}{2}+\frac{\alpha_2\eta}{2}+\left(\frac{3C^3}{125B^2}+\frac{5{\alpha_2}^2BD\kappa}{2C}+\frac{C\kappa u_2}{10B^{\frac{2}{3}}}\right)\tau-\frac{C\kappa\xi}{10B}\right], \quad (3)$$

or equivalently the following solutions for equation (1),

$$u(x,y,t)=\frac{6f^2\kappa}{25}+u_2+\frac{3f^2}{25}\mathrm{sech}^2\left[\frac{\alpha_4}{2}-\frac{\left(6f^4+625{\alpha_2}^2c\kappa+25f^2\kappa u_2\right)t}{250f}+\frac{f\kappa x}{10}+\frac{\alpha_2 y}{2}\right]+\frac{6f^2\kappa}{25}\tanh\left[\frac{\alpha_4}{2}-\frac{\left(6f^4+625{\alpha_2}^2c\kappa+25f^2\kappa u_2\right)t}{250f}+\frac{f\kappa x}{10}+\frac{\alpha_2 y}{2}\right], \quad (4)$$

where $\kappa=\pm1$ while u_2, α_2 and α_4 are all constants.

3 Comment 2

Since reference [1] does not mention any known solution-related transformation for equation (2), we need to address the importance of this issue.

For example, any auto-Bäcklund transformation for equation (2) would work as a system relating a known solution of equation (2) (e.g., expression (3)) to another solution of equation (2) itself. This way we would, in principle at least, be able to progressively construct more and more complicated solutions of equation (2).

In fact, the following auto-Bäcklund transformation [9] has been seen for equation (1), or equivalently, for equation (2),

$$u(x,y,t)=-\frac{12{\phi_x}^2}{\phi^2}+\frac{12\left(\frac{f\phi_x}{5}+\phi_{xx}\right)}{\phi}+u_2, \quad (5)$$

$$-25c{\phi_y}^2-25\phi_t\phi_x+f^2{\phi_x}^2-25u_2{\phi_x}^2-30f\phi_x\phi_{xx}+75{\phi_{xx}}^2-100\phi_x\phi_{xxx}=0, \quad (6)$$

$$cf{\phi_y}^2+f\phi_t\phi_x+5c\phi_{yy}\phi_x+fu_2{\phi_x}^2+5\phi_x\phi_{xt}+f^2\phi_x\phi_{xx}+5u_2\phi_x\phi_{xx}+6f\phi_x\phi_{xxx}+5\phi_x\phi_{xxxx}=0, \quad (7)$$

$$-cf\phi_{yy}\phi_x+2cf\phi_y\phi_{xy}-\frac{cf{\phi_y}^2\phi_{xx}}{\phi_x}=0, \quad (8)$$

$$(u_{2,t}+u_2u_{2,x}+u_{2,xxx})_x+cu_{2,yy}+fu_{2,xxx}=0, \quad (9)$$

where u_2 and ϕ are both analytic functions with $\phi_x\neq0$, and the whole set is mutually consistent.

Adding a few more words about Comment 1: solutions (3) and (4) come out from the substitution of our ansatzes,

$$\phi=1+e^{\alpha_4+\alpha_3t+\alpha_1x+\alpha_2y} \quad \text{and} \quad u_2=\text{constant},$$

into this Bäcklund transformation, where α_i's are all constants.

4 Comment 3

We also need to call the attention that equation (2) presented by reference [1] is a good model to give rise to certain two-dimensional and non-travelling-wave possibly *observable* effects [9] for the future space and laboratory plasma experiments, which reference [1] has not theoretically investigated as yet.

Beyond the travelling waves discussed in reference [1] and Comment 1, there exist some monotonic-shock-wave-like, exact analytic solutions for equation (1), or equivalently, for equation (2), derived via the aforementioned auto-Bäcklund transformation, as follows [9],

$$u(x,y,t)=-\frac{25c\alpha(t)^2}{f^2}+\frac{3}{25}f^2\mathrm{sech}^2\left[\frac{-\frac{f\sigma}{5}x+y\alpha(t)+\beta(t)}{2}\right]-\frac{6}{25}f^2\sigma\tanh\left[\frac{-\frac{f\sigma}{5}x+y\alpha(t)+\beta(t)}{2}\right]+\frac{5\sigma[y\alpha'(t)+\beta'(t)]}{f}, \quad (10)$$

where $\sigma=\pm1$, while $\alpha(t)$ and $\beta(t)$ are both real, differentiable functions.

Expression (10) provides us with several observable effects [9] which the future space and laboratory plasma experiments might discover, beyond the existing one-dimensional and travelling-wave considerations. The possibly observable effects are:

(1) the transverse disturbance characterized by the "$x\sim y$ slope" on the shock front;
(2) the non-constant velocities of propagation and non-constant vertical shifts of the shocks along the propagation direction (with y fixed);
(3) the non-constant-velocity transverse movements and non-constant vertical shifts of the shocks along the transverse direction (with x fixed).

More detailed and graphical discussions can be found in reference [9].

This work has been supported by the Excellent Young Teachers Program of the Ministry of Education of China, by the National Natural Science Foundation of China under Grants Nos. 10272017 and 60372095, by the National Key Basic Research Special Foundation (NKBRSF) of China under Grant No. G1999032701, by the W.T. Wu Foundation on Mathematics Mechanization, by the Talent Construction Special Fund of Beijing University of Aeronautics and Astronautics, by the Discipline Construction Grant BHB985-1-7 of Beijing University of Aeronautics and Astronautics under the Action Plan for the Revitalization of Education in the 21st Century of the Ministry of Education of China. BT also thanks the Enterprise Chair Professors Program of Beijing University of Posts and Telecommunications and the Bright Oceans Corporation. YTG would like to acknowledge the Cheung Kong Scholars Programme of the Ministry of Education of China and Li Ka Shing Foundation of Hong Kong.

References

1. J.-K. Xue, Eur. Phys. J. D **26**, 211 (2003)
2. Q. Luo, N. D'Angelo, R. Merlino, Phys. Plasmas **5**, 2868 (1998)
3. T. Takeuchi, S. Iizuka, N. Sato, Phys. Rev. Lett. **80**, 77 (1998)
4. Q. Luo, N. D'Angelo, R. Merlino, Phys. Plasmas **6**, 3455 (1999)
5. Y. Nakamura, H. Bailung, P. Shukla, Phys. Rev. Lett. **83**, 1602 (1999)
6. Y.T. Gao, B. Tian, Phys. Plasmas **8**, 3124 (2001)
7. P. Shukla, Phys. Plasmas **7**, 1044 (2000)
8. C. Liu, D. Zhang, Appl. Math.-JCU A **14**, 1 (1999) (in Chinese with English abstract)
9. Y.T. Gao, B. Tian, Phys. Plasmas **8**, 3146 (2001)
10. L. Molinet, J. Diff. Eqn. **152**, 30 (1999)
11. G.C. Das, J. Sarma, Phys. Plasmas **6**, 4394 (1999)
12. Z.Y. Yan, H.Q. Zhang, Phys. Lett. A **252**, 291 (1999); J. Phys. A **34**, 1785 (2001)
13. M. Senthilvelan, Appl. Math. Comput. **123**, 381 (2001)
14. E.G. Fan, J. Zhang, B.Y.C. Hon, Phys. Lett. A **291**, 376 (2001)
15. Sirendaoreji, J. Phys. A **32**, 6897 (1999)
16. E.G. Fan, Phys. Lett. A **277**, 212 (2000)
17. B. Li, Y. Chen, H.Q. Zhang, J. Phys. A **35**, 8253 (2002)
18. H.-N. Xuan, B. Li, Z. Naturforsch. A **58**, 167 (2003)
19. Y.T. Gao, B. Tian, Phys. Plasmas **10**, 4306 (2003); Y.T. Gao, B. Tian, Int. J. Mod. Phys. C **12**, 197 (2001); B. Tian, W. Li, Y.T. Gao, Acta Mech. **160**, 235 (2003); B. Tian, Y.T. Gao, Computers Math. Applic. **45**, 731, (2003); B. Tian, Y.T. Gao, Eur. Phys. J. B **22**, 351 (2001); Y.T. Gao, B. Tian, Phys. Plasmas **8**, 67 (2001); Y.T. Gao, B. Tian, Int. J. Mod. Phys. C **12**, 1431 (2001)
20. W.T. Wu, J. Sys. Sci. Math. Sci. **4**, 207 (1984); Kexue Tongbao **31**, 1 (1986)
21. X.S. Gao, Adv. Math. **30**(5), 385 (2001) (in Chinese)